Quasi-steady expansion of plasma ablated from laser-irradiated pellets

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The ablative, quasi-steady expansion of the spherical coronal plasma produced by irradiating an overdense pellet by a high-intensity laser pulse, is studied for large ion charge number Z_i. The entire structure of the flow and its changes as the laser power is increased, are determined. The instantaneous power W required to generate a given ablation pressure P_a and pellet radius r_a at any time is determined in terms of Z_i, ion mass m_i, and critical density; the mass ablation rate is also found. If the time law $P_a(t)$ and (consequently) $r_a(t)$ for a desired optimal compression of the pellet are determined independently, the results allow one to obtain the laser power history $W(t) = W[P_a(t), r_a(t), Z_i, m_i, n_c]$. For a critical radius much larger than r_a, most of the energy flows outward; this seems to invalidate known simple estimates of the relation $W(P_a)$.

I. INTRODUCTION

In order to achieve thermonuclear reactions by means of short, intense laser pulses, a pellet must be compressed to very high densities, and have its core heated to a high temperature.[1] This is only possible if the pressure at the pellet surface (the ablation pressure P_a) changes in time in a special manner, dependent on the type of pellet.[2] Clearly, the laser pulse should be appropriately shaped.

As a complement to the results of thorough numerical simulations,[3] it would be convenient to have available simple, analytical, scaling laws, relating P_a (and may be the mass ablation rate) to the laser power W, and to other parameters such as the critical density n_c (or equivalently the laser wavelength), the pellet radius r_a, the ion mass m_i, and charge number Z_i in the expanding corona of the plasma; this requires a careful analysis of the energy flux in the corona.

The planar problem has recently been studied, allowance being made for unsteady effects and different ion and electron temperatures, T_i and T_e, and using classical heat conduction, absorption at n_c, and a laser pulse linear in time, for which a self-similar motion develops[4-7]; the evolution of the behavior of both the coronal plasma and the dense pellet as the pulse rise-time shortens was thoroughly analyzed.

A steady, spherical corona was considered by Gitomer *et al.*,[8,9] who assumed $T_i = T_e$, absorption at n_c, and classical heat conduction; unsteady effects were allowed for in related numerical computations.[10] A scale law was given by Caruso[11] for the limit $r_c/r_a \to \infty$, r_c being the critical radius, where $n = n_c$.

In this paper we re-examine the work of Ref. 8 and complete it in four respects:

(i) We let T_e and T_i be different; the time for ion-electron energy relaxation is also the characteristic time for electron heat conduction in a quasi-neutral plasma with velocities comparable to the ion-acoustic speed. Having $T_e \neq T_i$ allows us to properly describe how the results depend on Z_i. Here, we assume Z_i large, decoupling T_i (which nevertheless remains *different* from T_e) from the rest of the problem; the finite Z_i case is considered in a companion paper.

(ii) We properly take into account that for the conditions of interest (a not too steep pulse initially, and n_c much less than the pellet density n_p)[5,6] there exists a well defined ablation surface; this closes the problem in a such a way that, instead of families of solutions as in Ref. 8 we arrive at a single universal relation between P_a and W, and the parameters r_a, n_c, m_i, and Z_i, the goal of the paper.

(iii) We find that for W less than a limiting value $W^l(r_a, n_c, m_i, Z_i)$, which we determine, the results of Ref. 8 are invalid. Our solution for $W < W^l$ is found to agree in the limit $W/W^l \to 0$ (when heat conduction is restricted to a thin layer) with planar results from Ref. 5.

(iv) We find a value $W^*(r_a, n_c, m_i, Z_i)$, at which the faraway plasma experiences a discontinuous transition in the way T_e decays wity radius r (from $T_e \sim r^{-4/3}$ for $W < W^*$, to $T_e \sim r^{-2/7}$ for $W > W^*$, through the transition law $T_e \sim r^{-2/5}$ for $W = W^*$). This phenomenon, similar to one found in planar, unsteady flow,[7] is of particular relevance for some effects not considered here, such as the appearance of a two-temperature electron population, the breakdown of quasi-neutrality, etc.

An important conclusion of our work is that when r_c/r_a is large, at the end of the compression, most of the energy absorbed at r_c flows outward; this result seems to invalidate simple estimates of the relation $P_a(W)$ in spherical geometry.[11,12]

II. STATEMENT OF THE PROBLEM

We consider a single ion-species plasma, and assume that the flow is quasi-neutral ($Z_i n_i \simeq n_e \equiv n$), spherically symmetric, and that the ratio n_c/n_p is small. To lowest order in an asymptotic expansion in that parameter, the density in the corona will go to infinity at the pellet surface, and the flow may be assumed steady [the pellet/corona characteristic velocity (and therefore the characteristic time) ratio is of order $(n_c/n_p)^{1/2}$ since the momentum fluxes in pellet and corona, being produced by P_a, are comparable].[5] The mass flow rate, being independent of r, and the pressure, being equal to P_a, should remain finite, hence, the ion and electron velocities (v_i and v_e) and temperatures, should vanish at the pellet. Clearly, the collision fre-

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quency ($\sim n/T_e^{3/2}$) approaches infinity at r_a, and there the heat flux must vanish and both T_i/T_e and v_i/v_e must approach unity. While temperatures may differ from each other outside the pellet, it may be shown, using quasi-neutrality, that $v_i \simeq v_e \equiv v$ throughout the corona. We use classical values[13] for the ion-electron energy relaxation time ($t_{ei} \equiv \bar{t}_{ei} T_e^{3/2}/n$) and the electron heat conductivity ($K_e \equiv \bar{K} T_e^{5/2}$), and neglect ion conduction and the weak variations of the Coulomb logarithm in both $\bar{t}_{ei}$ and $\bar{K}$.

In steady flow, mass conservation may be written in the form

$$nvr^2 = \mu \,, \tag{1}$$

where μ is a constant and $4\pi m_i \mu / Z_i$ is the mass flow rate. Using quasi-neutrality and neglecting electron inertia, the equation for momentum conservation in the ion-electron fluid reads

$$m_i n v \frac{dv}{dr} = -\frac{d}{dr}[nk(Z_i T_e + T_i)] \,, \tag{2}$$

where k is Boltzmann's constant. Finally, the electron and ion entropy equations are

$$nT_e v \frac{d}{dr}\left(k \ln \frac{T_e^{3/2}}{n}\right) = \frac{\bar{K}}{r^2}\frac{d}{dr}\left(r^2 T_e^{5/2}\frac{dT_e}{dr}\right) - \frac{3}{2}kn^2 \frac{T_e - T_i}{\bar{t}_{ei} T_e^{3/2}} + \frac{W\delta(r - r_c)}{4\pi r_c^2} \,, \tag{3}$$

$$\frac{n}{Z_i} T_i v \frac{d}{dr}\left(k \ln \frac{T_i^{3/2}}{n}\right) = \frac{3}{2}kn^2 \frac{T_e - T_i}{\bar{t}_{ei} T_e^{3/2}} \,; \tag{4}$$

absorption occurs at the critical radius given by

$$n(r_c) = n_c \,. \tag{5}$$

If Z_i is large, the ion pressure term may be dropped in (2), which, using (1), becomes

$$\frac{d}{dr}\left(\frac{1}{2}m_i v^2\right) = kZ_i T_e\left(\frac{2}{r} - \frac{1}{T_e}\frac{dT_e}{dr}\right)\left(1 - \frac{Z_i k T_e}{m_i v^2}\right)^{-1} . \tag{2$'$}$$

As seen from (4), the energy exchange term may also be dropped in (3) when Z_i is large; using (1) and (2′), and integrating from r_a to an arbitrary radius, we get

$$\mu\left(\frac{5}{2}kZ_i T_e + \frac{1}{2}m_i v^2\right) = Z_i \bar{K} r^2 T_e^{5/2}\frac{dT_e}{dr} + \frac{WZ_i H(r)}{4\pi} \,, \tag{3$'$}$$

where H is the Heaviside function

$$H = 0 \ (r < r_c), \quad H = 1 \ (r > r_c) \,;$$

to obtain (3′), which is the energy equation for the ion-electron fluid, we used the conditions $v = T_e = T_e^{5/2} dT_e / dr = 0$ at $r = r_a$. Equation (5) becomes

$$\mu = r_c^2 v(r_c) n_c \,.$$

We must then solve Eqs. (2′) and (3′), subject to the boundary conditions

$$\left.\begin{aligned} &T_e = 0 \qquad (6)\\ &\mu k T_e / v r^2 = P_a \qquad (7)\end{aligned}\right\} \text{at } r = r_a \,,$$

where the pressure, nkT_e, was rewritten by means of (1), in the form $\mu k T_e / r^2 v$. An additional boundary condition is that as $r \to \infty$, T_e becomes neither negative nor multivalued; this, together with the requirement of vanishing density (and pressure) at infinity, is seen to imply

$$T_e \to 0 \text{ as } r \to \infty \,. \tag{8}$$

Finally, for v not to be multivalued, either the numerator on the right-hand side of (2′) must vanish at the isothermal sonic point or this point lies at the critical radius: either

$$\frac{2}{r} - \frac{1}{T_e}\frac{dT_e}{dr} = 0 \,, \tag{9a}$$

or

$$r_c = r_s \,, \tag{9b}$$

at

$$r = r_s (v_s^2 = Z_i k T_{es} / m_i) \,.$$

Equations (6)–(9) completely determine the solution of the two first-order equations (2′) and (3′), which, for given P_a (and n_c, r_a, m_i, Z_i), involve two eigenvalues, μ and W. Once $v(r)$ and $T_e(r)$ have been found, n and T_i follow from (1), (4), and the boundary condition

$$T_i / T_e \to 1 \text{ as } r \to r_a \,. \tag{10}$$

We now introduce the dimensionless variables

$$\eta = r/r_a, \quad \theta = T_e/T_r, \quad u = v/(Z_i k T_r / m_i)^{1/2} \,; \tag{11}$$

then, Eq. (2′) becomes

$$\frac{1}{2}\left(1 - \frac{\theta}{u^2}\right)\frac{du^2}{d\eta} = \frac{2\theta}{\eta} - \frac{d\theta}{d\eta} \,. \tag{12}$$

We choose the reference temperature T_r in such a way that condition (7) reads "$\theta/u = 1$ at $\eta = 1$"; then, we have

$$T_r = \frac{Z_i}{m_i k}\frac{P_a^2 r_a^4}{\mu^2} \,, \tag{13}$$

and Eq. (3′) becomes

$$\frac{5}{2}\theta + \frac{1}{2}u^2 = \beta \eta^2 \theta^{5/2}\frac{d\theta}{d\eta} + \bar{W} H(\eta) \,, \tag{14}$$

where

$$H = 0 \ (\eta < \eta_c \equiv r_c / r_a), \quad H = 1 \ (\eta > \eta_c) \,,$$

$$\beta = \left(\frac{Z_i}{m_i}\right)^{5/2}\frac{\bar{K}}{k^{7/2}}\frac{r_a^{11} P_a^5}{\mu^6} \,, \tag{15}$$

$$\bar{W} = \frac{m_i}{4\pi Z_i}\frac{\mu W}{r_a^4 P_a^2} \,, \tag{16}$$

and η_c is given by

$$\eta_c^2 u(\eta_c) = \frac{m_i}{Z_i n_c}\frac{\mu^2}{P_a r_a^4} \,. \tag{17}$$

The boundary conditions are

$$\left.\begin{aligned} &\theta = 0 \qquad (18)\\ &u/\theta = 1 \qquad (19)\end{aligned}\right\} \text{at } \eta = 1 \,,$$

$$\theta \to 0 \text{ as } \eta \to \infty \,, \tag{20}$$

either

$$\frac{2}{\eta} - \frac{d \ln \theta}{d\eta} = 0 \,, \tag{21a}$$

or

$$\eta_c = \eta_s \tag{21b}$$

at η_s ($u_s^2 = \theta_s$). The equation for $\theta_i \equiv T_i/T_r$ is

$$\beta\eta^2\left(\frac{3}{2}u^2\frac{d\theta_i}{d\eta} + \frac{2u^2\theta_i}{\eta} + \frac{1}{2}\theta_i\frac{du^2}{d\eta}\right) = 41.7\frac{\theta_e - \theta_i}{\theta_e^{3/2}},$$

$$\theta_i/\theta \to 1 \quad \text{as } \eta \to 1.$$

In the next section we solve these equations choosing η_c as a free parameter in the range $1 < \eta_c < \infty$; for each η_c we obtain β, $\bar{W}$, and $u(\eta_c)$, and then, from (15)–(17), we get W, P_a, and μ, leading to the relations $W(P_a, r_a, n_c, m_i, Z_i)$ [or $P_a(W, r_a, n_c, m_i, Z_i)$] and $\mu(W, r_a, n_c, m_i, Z_i)$.

III. GENERAL INTEGRATION OF THE EQUATIONS

Two points should be noted prior to any further development; (i) For $H=0$ ($\eta < \eta_c$), Eqs. (12) and (14) allow us to introduce phase-plane variables

$$Y = \eta\beta\theta^{5/2}, \quad M^2 = u^2/\theta \tag{22}$$

(M is the Mach number based on the isothermal sonic speed) obeying the equations

$$\frac{dY}{d\ln\eta} = Y + \frac{5(M^2+5)}{4}, \tag{23}$$

$$dM^2/d\ln\eta = M^2[8Y - (M^2+5)(M^2+1)][2Y(M^2-1)]^{-1}, \tag{24}$$

and eliminating η,

$$\frac{dM^2}{dY} = 2M^2[8Y - (M^2+5)(M^2+1)] \times [Y(M^2-1)(4Y+5M^2+25)]^{-1}. \tag{25}$$

(ii) As pointed out in Ref. 8, η_c cannot be less than η_s: if $\eta_c \neq \eta_s$, we must have [see Eq. (21a)] $d\theta/d\eta = 2\theta/\eta > 0$ at η_s, while $d\theta/d\eta$ should be negative beyond η_c. Hence, (23)–(25) are valid at least up to the sonic radius.

A. Absorption beyond the sonic radius ($\eta_c > \eta_s$)

Equations (18) and (19) imply $Y=0, M^2=0$ at $\eta=1$. The solution to (25) near the point $Y=0, M^2=0$, which is a nodal point, is $M^2 \simeq CY^{2/5}$; the constant C is related to the eigenvalue β through the boundary condition (19). We then get

$$M^2 = (Y/\beta)^{2/5}, \quad Y \ll 1. \tag{26}$$

At the sonic point ($M^2=1$), dM^2/dY must remain finite to avoid a multivalued solution; that is, the numerator on the right-hand side of (25) must vanish [condition (21a)], leading to $Y=3/2$. There is a value of β (call it β_l) that allows solution (26) to reach the (saddle) point $Y=3/2, M^2=1$ (see Fig. 1). Starting at this point and integrating backward toward the node we numerically find $\beta_l \simeq 11.30$. Beyond the point $Y=3/2$, the solution $M^2(Y)$ is valid up to the critical radius, wherever that may be. Once $M^2(Y)$ is known, Eq. (23) with the boundary condition $Y=0$ at $\eta=1$ yields $Y(\eta)$, and then $u(\eta)$ and $\theta(\eta)$, for $\eta < \eta_c$. They are given, together with θ_i in Fig. 2. The sonic point occurs at $\eta_s = 1.215 \equiv \eta_{sl}$. A simple estimate of η_{sl} may be obtained by noting that for $1 < \eta \leq \eta_{sl}$, we have $0 < u^2/\theta \leq 1$ and therefore

$$\beta\eta^2\theta^{5/2}\left(\frac{d\theta}{d\eta}\right) = \alpha\theta, \quad \left(\frac{5}{2} < \alpha \equiv \frac{5}{2} + \frac{u^2}{2\theta} < 3\right)$$

yielding, for some intermediate α,

$$\beta\theta^{5/2} = 5\alpha(\eta-1)/\eta;$$

at the sonic point we also have $3\theta = (\beta\eta^2\theta^{5/2})2\theta/\eta$, so that finally $1.20 < \eta_{sl} = 1 + 3/5\alpha < 1.24$.

For any chosen value of $\eta_c > \eta_{sl}$, Fig. 2 yields both $u(\eta_c)$ and $\theta(\eta_c)$. Equations (12) and (14) could then be integrated, starting at η_c, if the eigenvalue $\bar{W}$ were known; $\bar{W}$ is determined by the boundary condition (20). To carry out the numerical integration it is actually more convenient to start at large η and integrate backward.

Let us then consider the behavior of the solution for large η, well beyond η_c. It was found in Ref. 8 that θ could decay either as $\theta \sim \eta^{-4/3}$ (adiabatic flow) or $\theta \sim \eta^{-2/7}$, this second case seeming to be the real one for large enough η_c. We find however that; (i) there exists

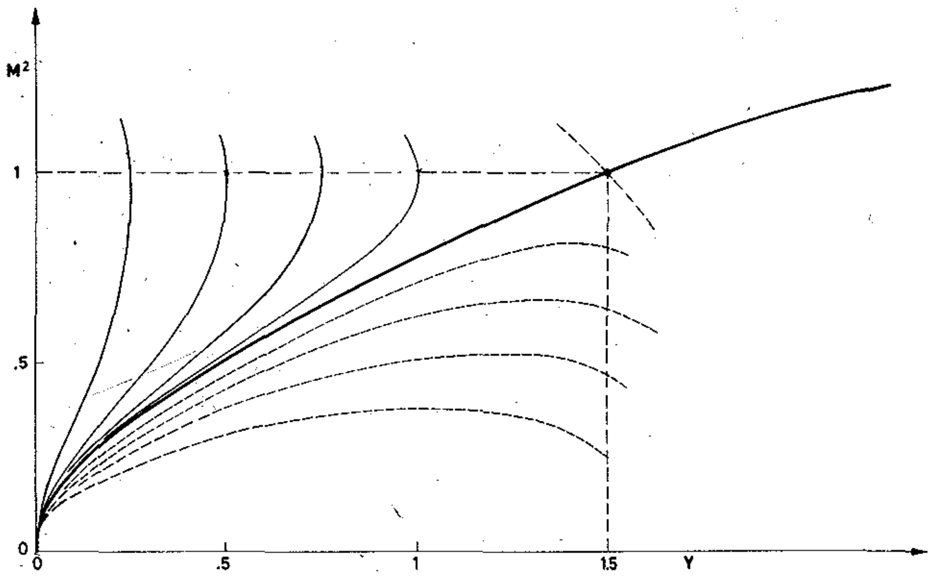

FIG. 1. Integral curves in the phase-plane $M^2 - Y$ (dashed curves are schematic).

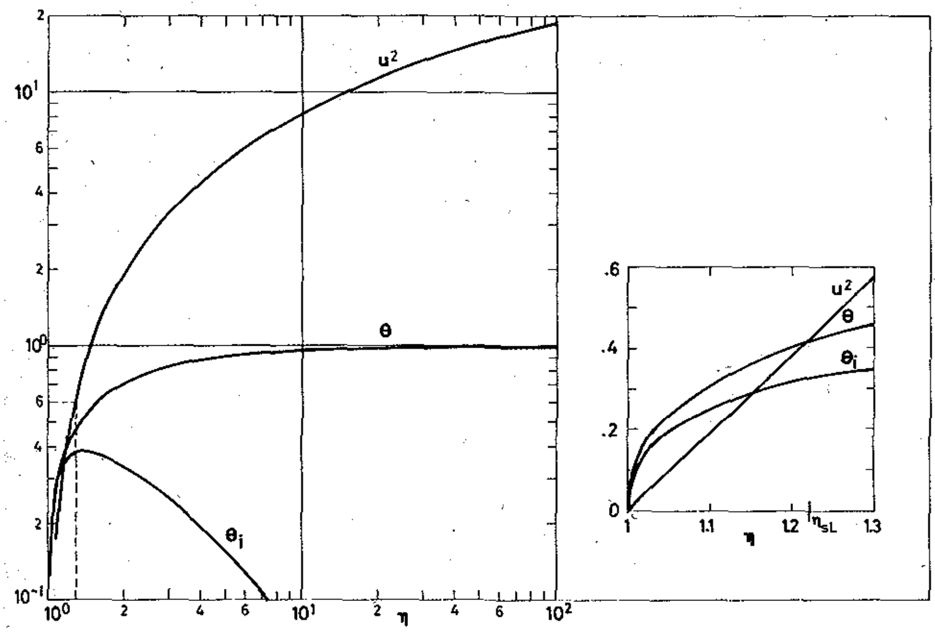

FIG. 2. Dimensionless squared velocity and temperatures versus dimensionless radius for $1 \le \eta \le \eta_c$ and $\eta_{sl} \le \eta_c$.

a third decay law, $\theta \sim \eta^{-2/5}$, and (ii) there is a transition value η_c^* such that if $\eta_c < \eta_c^*$, then

$$\theta \simeq C_1 \eta^{-4/3}, \quad u^2 - 2\bar{W} \simeq -5\theta ; \qquad (27)$$

if $\eta_c > \eta_c^*$,

$$\theta \simeq C_2 \eta^{-2/7}, \quad u^2 - 2\bar{W} \simeq -4\beta_i C_2^{7/2}/7 - 16\theta; \qquad (28)$$

if $\eta_c = \eta_c^*$,

$$\theta \simeq (35/4\beta_i \eta)^{2/5}, \quad u^2 - 2\bar{W}^* \simeq -12\theta , \qquad (29)$$

where $\bar{W}^* = \bar{W}(\eta_c^*)$. We find numerically that $\eta_c^* \simeq 2.05$, $\bar{W}^* \simeq 5.55$. If η_c is less than, but close to η_c^*, C_1 is large and the approximation (27) breaks down for $\eta = O(C_1^{15/14})$, the solution taking the form (29) for $\eta_c \ll \eta \ll C_1^{15/14}$. Similarly for η_c larger than, but close to η_c^*, C_2 is small, and (28) breaks down for $\eta = O(C_2^{-35/4})$, (29) being valid for $\eta_c \ll \eta \ll C_2^{-35/4}$. The regions of validity of (27) and (28) are removed to infinity as $\eta_c - \eta_c^* \to 0^-$ ($C_1 \to \infty$) and $\eta_c - \eta_c^* \to 0^+$ ($C_2 \to 0$), respectively. More details of the transition are discussed in the Appendix.

Then, to perform the integration for the range $\eta > \eta_c$, one chooses some arbitrary value of C_1 in (27), and sweeps over a convenient $\bar{W}$ range until a value is found for which $\theta(\eta)$ and $u(\eta)$ meet the curves in Fig. 2 at a common η, which will be η_c. Similar procedures are followed to use (28) and (29).

B. Absorption at the sonic radius ($\eta_c = \eta_s$).

The eigenvalue $\bar{W}$ decreases with decreasing η_c. When $\eta_c = \eta_{sl} \simeq 1.215$, $\bar{W}$ takes the limiting value $\bar{W}^l \simeq 1.86$. Since η_c cannot be less than η_s, one must have $\eta_c = \eta_s < \eta_{sl}$ for $\bar{W} < \bar{W}^l$. Then, to obtain the solution for

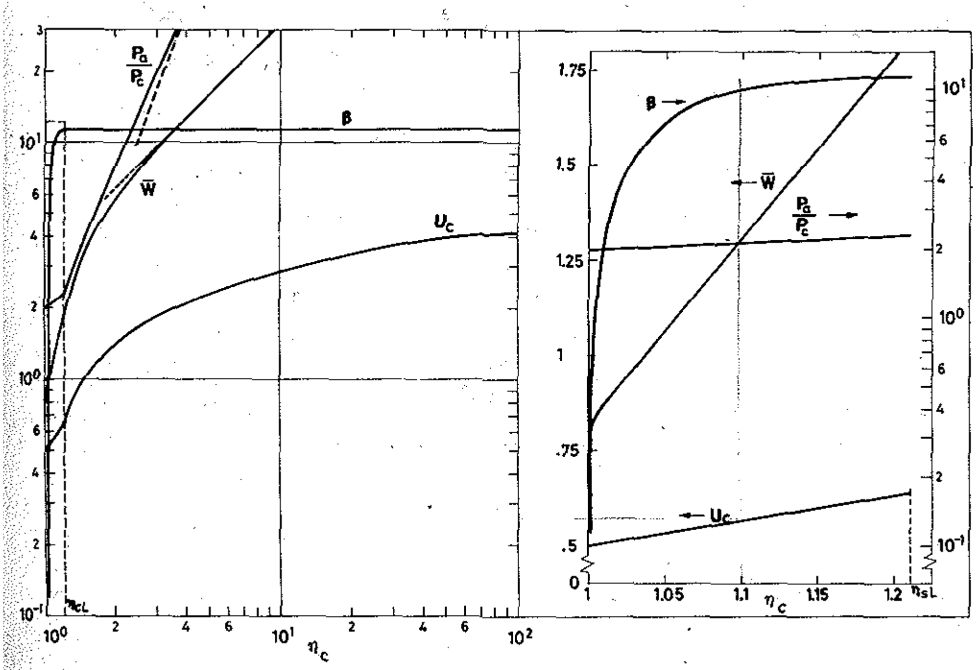

FIG. 3. Dimensionless parameters β, $\bar{W}$, and critical velocity, and ablation to critical pressure ratio versus critcal radius (dashed curves are the asymptotic behavior for large η_c).

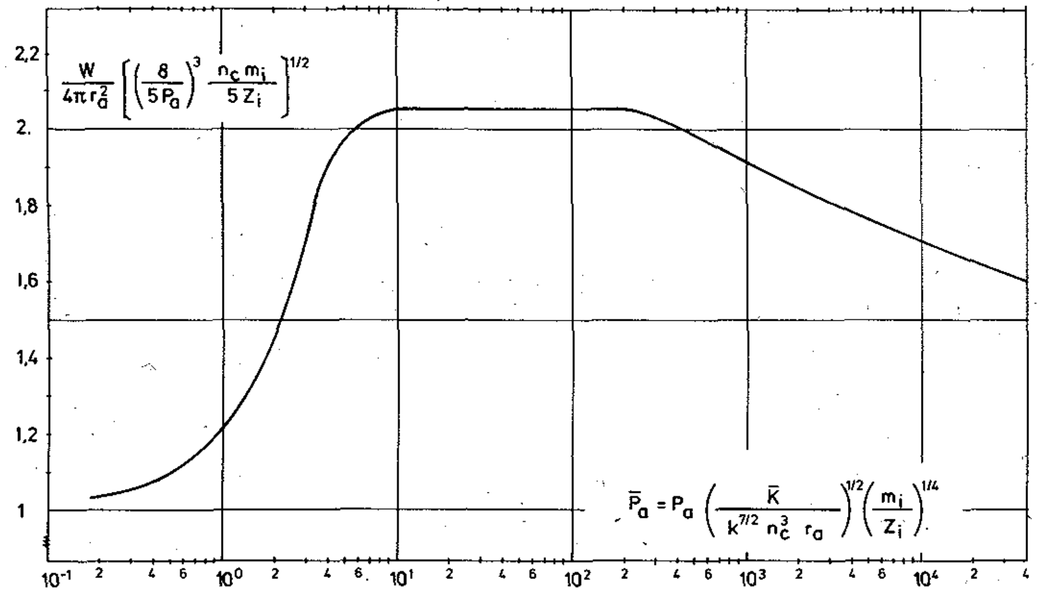

FIG. 4. Laser power versus ablation pressure.

this range, one chooses an arbitrary sonic point $M^2=1$, $Y<3/2$ (see Fig. 1) and integrates toward the node $Y=0$, $M^2=0$; in this way, a value $\beta<\beta_l$ is determined in (26). Once $M^2(Y)$ is known, Eq. (23) may be solved, yielding $u(\eta)$, $\theta(\eta)$ for $\eta \leqslant \eta_s$, and a value $\eta_s < \eta_{sl}$ is found. Since now $\eta_c = \eta_s$, the integration beyond η_s proceeds as in Sec. III A.

Complete results for $\beta(\eta_c)$, $\overline{W}(\eta_c)$, and $u(\eta_c)$ are given in Fig. 3. Also shown for later reference is the ratio of ablation pressure P_a to critical pressure $P_c \equiv n_c k T_e(r_c)$. From Fig. 3 and Eqs. (15)–(17), P_a, W, and μ can be obtained as functions of η_c. Finally, we arrive at $W(P_a)$ and $\mu(P_a)$, shown in Figs. 4 and 5; for convenience, $P_a(W)$ is shown in Fig. 6.

IV. THE LOW-POWER ($\eta_c \rightarrow 1$) LIMIT

At the beginning of the pulse, the critical surface will lie close to the pellet, that is, $\eta_c \simeq 1$. Figure 3 shows that $\beta \rightarrow 0$ as $\eta_c \rightarrow 1$; as seen in Eqs. (12) and (14) heat conduction is restricted, for small β, to a thin layer where the gradients are large, the flow outside being isentropic:

$$\frac{1}{2}\left(1-\frac{\theta}{u^2}\right)\frac{du^2}{d\eta}=\frac{2\theta}{\eta}-\frac{d\theta}{d\eta}\,, \tag{30}$$

$$\tfrac{5}{2}\theta+\tfrac{1}{2}u^2=\overline{W}. \tag{31}$$

The solution to Eqs. (30) and (31) is

$$u^{2/3}(2\overline{W}-u^2)=5A\eta^{-4/3}\,, \tag{32}$$

$$\theta(2\overline{W}-5\theta)^{1/3}=A\eta^{-4/3}\,, \tag{33}$$

where A is a constant. It should be noted that the flow velocity can be less than the isentropic sound speed ($u^2<5\theta/3$) nowhere in this region because, otherwise, from

$$5\frac{d\theta}{d\eta}=-\frac{du^2}{d\eta}=-\frac{(20\theta/3\eta)u^2}{u^2-5\theta/3}\,, \tag{34}$$

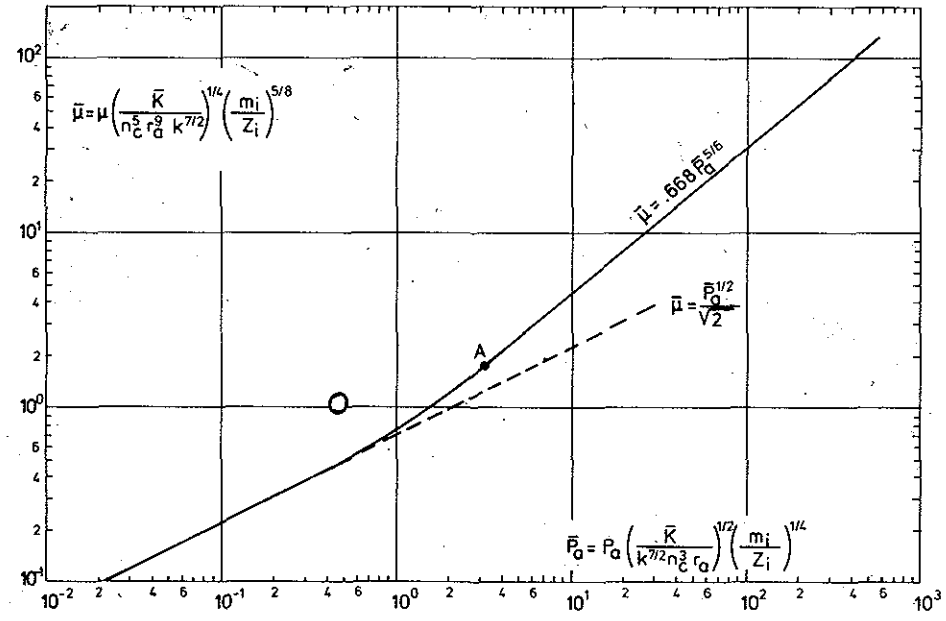

FIG. 5. Mass ablation rate versus ablation pressure. To the right of point A ($\eta_c = \eta_{sl}$), the simple law shown holds. Dashed line is the asasymptotic behavior for small ablation pressure.

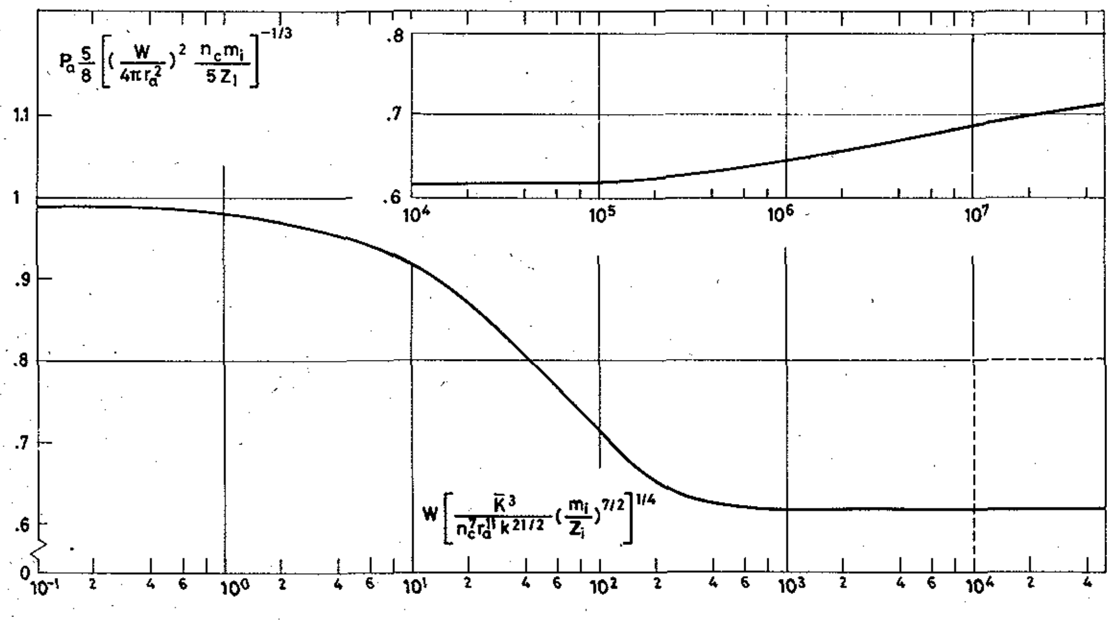

FIG. 6. Ablation pressure versus laser power.

which follows from (30) and (31), we would get a positive $d\theta/d\eta$ value. We thus have a supersonic and isentropic expansion to vacuum, that can only start from sonic or supersonic conditions.

In the thin, nonisentropic layer, the first term on the right-hand side of (30), arising from spherical geometry, may be dropped against the second one. An immediate integration then yields

$$u^2+\theta=u\,, \tag{35}$$

where the condition $\theta/u=1$ at $\eta=1$ has been used. One easily verifies that both u and θ are of order unity throughout the layer; hence, its width will be of the order of β. Defining $\hat{\eta}=(\eta-1)/\beta$, Eq. (14) becomes

$$\theta^{5/2}\frac{d\theta}{d\hat{\eta}}=\frac{5\theta}{2}+\frac{u^2}{2}-\overline{W}H\,. \tag{36}$$

Equations (35) and (36) have been studied in Ref. 7. The solution to (35) valid up to terms of order of β, is

$$u=\tfrac{1}{2}[1\pm(1-4\theta)^{1/2}]\,. \tag{37}$$

Since the maximum of θ must lie at the critical radius, the lower (upper) sign corresponds to $\hat{\eta}<\hat{\eta}_c$ $(\hat{\eta}>\hat{\eta}_c)$, and $\theta_c=1/4$, $u_c=1/2$ (Ref. 7); the flow is indeed (isothermally) sonic at the critical surface. Using (37) in (36) we get

$$\hat{\eta}=\int_0^\theta x^{5/2}\left(\frac{5}{2}x+\frac{1}{8}[1\pm(1-4x)^{1/2}]^2-\overline{W}H\right)^{-1}dx\,. \tag{38}$$

The value $\hat{\eta}_c=\hat{\eta}(1/4)\simeq 4.66\times10^{-3}$ is obtained from (38) with the lower sign and $H\equiv0$. The function $F(\theta)=5\theta/2+\{[1+(1-4\theta)^{1/2}]^2\}/8$ presents a maximum 25/32 at $\theta=15/64$. On the other hand, the denominator of (38) must vanish when $\hat{\eta}\to\infty$; hence, we should have $24/32\equiv F(1/4)\leq\overline{W}\leq F(15/64)=25/32$. This inequality, together with (37), leads to the condition that u^2/θ lie between 5/3 and unity when $\hat{\eta}\to\infty$. Matching the inner $(\hat{\eta}\to\infty)$ and the outer $(\eta-1\to0)$ solutions for u^2/θ then yields $u^2/\theta=5/3$, that is, the Chapman–Jouguet condition (sonic flow) behind the thin conduction layer is satisfied[7]; also, we find $\overline{W}=25/32$.

Using (16) and (17), together with $\eta_c\simeq1$, $u(\eta_c)\simeq1/2$, $\overline{W}=25/32$, we finally arrive at

$$\mu=(Z_i n_c/2m_i)^{1/2}P_a^{1/2}r_a^2\,,$$

$$W=4\pi r_a^2(25/32)(2Z_i/n_c m_i)^{1/2}P_a^{3/2}\,.$$

The inner (subsonic) and the outer (supersonic) solutions can be matched smoothly, analyzing an intermediate layer, at the transonic region, where $\hat{\eta}\gg1$ but $\eta-1\ll1$; therefore, we shall take $\xi=\epsilon(\beta)\cdot\hat{\eta}$ of order unity with $\epsilon(\beta)\ll1$, and thus, $\eta-1=(\beta/\epsilon)\xi$ with $\beta/\epsilon\to0$. The dependent variables θ and u^2 and the eigenvalue $\overline{W}$ differ from their isentropic sonic values, 15/64, 25/64, and 25/32, respectively, in terms of the order of ϵ. Defining the expansions in powers of ϵ

$$\theta=(15/64)(1+\epsilon\theta_1+\epsilon^2\theta_2+\cdots)\,,$$

$$u^2=(25/64)(1+\epsilon u_1+\epsilon^2u_2+\cdots)\,,$$

$$\overline{W}=(25/32)(1+\epsilon\overline{W}_1+\epsilon^2\overline{W}_2+\cdots)\,,$$

and, by substitution into Eqs. (12) and (14), we get

$$\theta_1+u_1/3=4\overline{W}_1/3\,, \tag{39}$$

for the terms of order ϵ in both equations (the order unity terms reduce to an identity), and

$$\theta_2+u_2/3+3\theta_1^2-6\overline{W}_1\theta_1-2\xi=C_3\,, \tag{40}$$

$$\theta_2+\frac{u_2}{3}=\frac{2}{5}\left(\frac{15}{64}\right)^{5/2}\frac{d\theta_1}{d\xi}+\frac{10\overline{W}_2}{3}\,, \tag{41}$$

for the terms of order ϵ^2, respectively. It can easily be shown, from Eq. (12), that $\epsilon=\beta^{1/3}$ (the small parameter of the asymptotic theory of transonic flow[14]). The constants $\overline{W}_1$ and C_3 can be obtained from the matching of (39) and (40), for $\xi\to0$, with the solution (35) (independent of $\hat{\eta}$ and valid up to terms of order β) thereby obtaining $\overline{W}_1=C_3\equiv0$. The remaining equation

$$\frac{d\theta_1}{d\xi}+\left(\frac{15}{2}\right)\left(\frac{64}{15}\right)^{5/2}\theta_1^2-5\left(\frac{64}{15}\right)^{5/2}\left(\xi-\frac{2\bar{W}_2}{3}\right)=0\,, \tag{42}$$

which follows from (40) and (41), must have the asymptotic behavior $\theta_1\to\infty$ for $\xi\to 0$, and $d\theta_1/d\xi\to 0$ for $\xi\to\infty$, in order to match correctly with the subsonic and supersonic regions, respectively. These two conditions provide the value of the integration constant and the eigenvalue $\bar{W}_2$. Solution of (42) can be carried out in terms of the Airy functions giving $\bar{W}_2=9.34\times 10^{-2}$, and then $\bar{W}=(25/32)[1+3.345(\eta_c-1)^{2/3}+\cdots]$, in terms of η_c instead of β, using the $\hat{\eta}_c$ value.

V. THE HIGH-POWER ($\eta_c\to\infty$) LIMIT

Near the end of the compression the pellet radius is expected to become a small fraction of the critical radius, that is, $\eta_c\to\infty$. In this limit, some simple analytical results may be obtained.

To determine $u(\eta_c)$ and $\theta(\eta_c)$ for large η_c it will suffice to obtain the asymptotic solution to Eqs. (23)–(25) for large η. Since θ increases up to η_c, it is clear that $Y\to\infty$ as $\eta\to\infty$ (see Fig. 2). In addition, it is easily shown that $M^2/Y\to 0$ as $Y\to\infty$; in fact, we have roughly $M^2\sim\ln Y$. Therefore, in the lowest approximation, Eq. (23) becomes $dY/d\ln\eta\simeq Y$, which implies $Y/\eta=\text{const}$. Then, Eq. (22) yields $\theta\to\text{const}\equiv\theta_\infty$ as $\eta\to\infty$. On the other hand, (25) becomes $dM^2/dY\simeq 4M^2/[Y(M^2-1)]$, whose solution is $4\ln Y-M^2+\ln M^2=\text{const}$; using (22) and $\theta\to\theta_\infty$ we arrive at $u^2-\theta_\infty\ln u^2-4\theta_\infty\ln\eta=\text{const}$; the constant being -3.94 and θ_∞ being 0.992, found numerically.

The approximation $u^2\simeq 4\theta_\infty\ln\eta$, used in Ref. 8, is only valid for η unrealistically large; in that case one finds, for the next approximation, $\theta\simeq\theta_\infty-(2\ln\eta)/(\eta\beta_i\theta_\infty^{3/2})$. It is clear therefore that for η_c large, and neglecting terms of order $\eta_c^{-1}\ln\eta_c$ against unity, we may set

$$\theta_c\simeq\theta_\infty\simeq 0.992\,,$$

$$u_c^2-\theta_\infty\ln u_c^2-4\theta_\infty\ln\eta_c\simeq\text{const}\simeq -3.94\,. \tag{43}$$

To analyze the range $\eta>\eta_c$, for η_c large, we define $\bar{\eta}=\eta/\eta_c$, $\bar{\theta}=\theta/\theta_c$, $\bar{u}=u/u_c$, $u_c^2/\theta_c=M_c^2$. Then, from (12) and (14), we get

$$(M_c^2-\bar{\theta}/\bar{u}^2)\frac{d\bar{u}^2}{d\bar{\eta}}=\frac{4\bar{\theta}}{\bar{\eta}}-\frac{2d\bar{\theta}}{d\bar{\eta}}\,, \tag{44}$$

$$\frac{(5\bar{\theta}+M_c^2\bar{u}^2)}{\eta_c}=\frac{2\bar{W}}{\theta_c\eta_c}+\frac{4}{7}\beta_i\theta_c^{5/2}\bar{\eta}^2\frac{d(\bar{\theta}^{7/2})}{d\bar{\eta}}\,. \tag{45}$$

Notice that $M_c^2\sim\ln\eta_c$, for η_c large. We formally consider M_c^2 and η_c independent parameters and expand the solution to (44) and (45) in powers of η_c^{-1}, retaining the entire dependence on M_c^2: $\bar{\theta}=\bar{\theta}_0(\bar{\eta})+\eta_c^{-1}\theta_1(\eta)+\cdots$, $\bar{u}^2=\bar{u}_0^2(\bar{\eta})+\eta_c^{-1}\bar{u}_1^2(\bar{\eta})+\cdots$, $\bar{W}/\eta_c=\bar{W}_0+\eta_c^{-1}\bar{W}_1+\cdots$, obtaining successively

$$\frac{2}{7}\beta_i\theta_c^{5/2}\bar{\eta}^2\frac{d(\bar{\theta}_0^{7/2})}{d\bar{\eta}}+\frac{\bar{W}_0}{\theta_c}=0,\quad \bar{\theta}_0(1)=1,\quad \bar{\theta}_0(\infty)=0\,,$$

$$\frac{M_c^2-\bar{\theta}_0}{\bar{u}_0^2}\frac{d\bar{u}_0^2}{d\bar{\eta}}=\frac{4\bar{\theta}_0}{\bar{\eta}}-\frac{2d\bar{\theta}_0}{d\bar{\eta}}\,,\quad \bar{u}_0^2(1)=1\,,$$

$$\beta_i(\theta_c\bar{\theta}_0)^{5/2}\bar{\eta}^2\frac{d\bar{\theta}_1}{d\bar{\eta}}+\frac{\bar{W}_1}{\theta_c}=\left(\frac{5\theta_0+M_c^2\bar{u}_0^2}{2}\right)$$

$$\bar{\theta}_1(1)=\bar{\theta}_1(\infty)=0\,.$$

The equations for $\bar{\theta}_0$, $\bar{\theta}_1$, etc., which are of first order must satisfy two boundary conditions each; this allows us to determine $\bar{W}_0$, $\bar{W}_1$, ..., etc. Proceeding straightforwardly we arrive at

$$\bar{\theta}_0=\bar{\eta}^{-2/7},\quad \bar{W}_0=(2/7)\beta_i\theta_c^{7/2}\,,$$

$$\bar{\eta}^{-2/7}=[(M_c^2/15)+1]\bar{u}_0^{1/8}-(M_c\bar{u}_0)^2/15$$

$$\bar{\theta}_1=\frac{\bar{\eta}^{5/7}}{\beta_i\eta_c\theta_c^{5/2}}\int_1^{\bar{\eta}}\left(\frac{5}{2x^{2/7}}+\frac{M_c^2\bar{u}_0^2(x)}{2}-\frac{\bar{W}_1}{\theta_c}\right)\frac{dx}{x^2}\,,$$

$$\bar{W}_1=\theta_c\int_1^\infty\left(\frac{5}{2\bar{\eta}^{2/7}}+\frac{M_c^2u_0^2}{2}\right)\frac{d\bar{\eta}}{\bar{\eta}^2}\equiv\theta_c\frac{M_c^2}{2}f(M_c^2)$$

$[f(M_c^2)\to 1$ as $M_c^2\to\infty]$. Hence, we get

$$\bar{W}\simeq(2/7)\beta_i\eta_c\theta_c^{7/2}+(u_c^2/2)f(M_c^2)+\cdots\,, \tag{46}$$

θ_c, u_c and $M_c^2\equiv u_c^2/\theta_c$ being given by (43). This asymptotic behavior is shown in Fig. 3.

Retaining only the first term in (46), and eliminating μ between (16) and (17) we arrive at

$$W\simeq 4\pi r_a^2\left(\frac{Z_iP_a^3}{m_in_c}\right)^{1/2}\left(\frac{2\beta_i\theta_c^{7/2}}{7u_c^{1/2}}\right).$$

Note that u_c depends on η_c, and therefore (15) must be used together with (16) and (17) to entirely eliminate η_c and μ; this introduces the constant $\bar{K}$ in the relation $P_a(W)$ or $W(P_a)$. However, u_c depends very weakly on η_c; therefore, the ablation pressure is practically independent of $\bar{K}$, as in planar, steady flow.[5]

VI. DISCUSSION

We have studied the ablative, steady expansion of the spherical coronal plasma produced by irradiating an overdense pellet by a high-intensity laser pulse. We assumed quasi-neutrality, absorption at the critical density, classical heat conduction, and large ion charge number, and let the ion and electron temperatures differ from each other. We have found a universal relation, shown in Fig. 4 (and Fig. 6 too), between ablation pressure P_a, laser power W, pellet radius r_a, critical density n_c, ion mass m_i, and charge number Z_i. We also found the mass ablation rate in terms of these parameters (Fig. 5).

For a given target, one may determine, by neglecting the mass ablation rate, the time law for the ablation pressure, $P_a(t)$, that generates an optimal compression; this will be called the inner problem and has been studied elsewhere.[15] Its solution yields the pellet radius as a function of time, $r_a(t)$, too. The present paper solves what we will call the outer problem: Considering the expansion flow to be quasi-steady (a good approximation for n_c much less than pellet density), we determine for given, instantaneous, P_a and r_a, the laser power required $W(P_a,r_a,n_c,Z_i,m_i)$. Hence, for any pellet, we can, using Fig. 4, obtain the laser power history that generates an optimal compression

$$W(t)=W[P_a(t),r_a(t),n_c,Z_i,m_i]\,.$$

Our solution provides the small mass ablation rate (Fig. 5), which can then be used to obtain a corrected solution for the inner problem, if desired; for powers high enough to yield $r_c/r_a \geq 1.215$ (r_c is the critical radius), the mass ablation rate takes a particularly simple form (see Fig. 5).

For $r_c/r_a < 1.215$, we find the flow sonic at r_c. In the model considered here, in which absorption occurs at a surface, the sonic condition leads to infinite acceleration; in reality, absorption may occur in a thin layer around the critical surface, and large accelerations will result. *A consequence of this, is a fast drop in density* in that region. The ions, which are being heated by collisions with electrons, will then experience a sudden cooling. This is clearly seen in Fig. 7, where T_i first increases with radius, cools suddenly around the sonic critical surface, and then heats up again.

It should be noted that in steady flow the plasma velocity far from the pellet approaches a constant, and is only a function of the instantaneous value of W (or P_a). For fast rising laser pulses, as required for the isentropic compression of homogeneous pellets, the plasma *ablated at late times in the pulse may catch up with* plasma ablated earlier; a (collisionless) shock may thus form in the low-density corona.

The use of classical heat conduction limits the validity *of the results to moderate powers.* In this respect we found that below a certain power W^*, the temperature T_e drops fast beyond the critical surface, while above W^* the decay is much slower. Since the mean-free-path is proportional to T_e^2, the plasma beyond r_c, for $W > W^*$, should soon become collisionless, We find that for $W = W^*$, the laser intensity $\phi = W/4\pi r_c^2$ takes the value

$$\phi^* \simeq 95\left(\frac{k^3}{m_p}\right)^{7/8}\left(\frac{Z_i}{A_i}\right)^{7/8}\frac{n_c^{7/4} r_c^{3/4}}{[\bar{K}(Z_i)]^{3/4}},$$

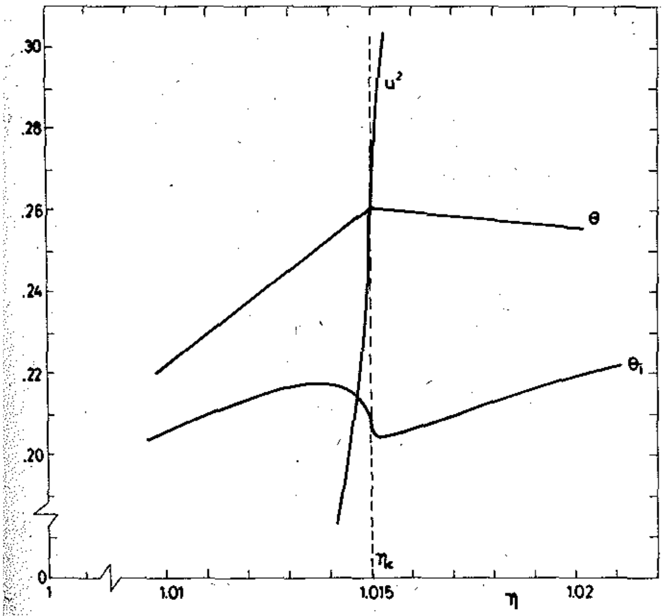

FIG. 7. Dimensionless squared velocity and temperatures versus dimensionless radius for a given critical radius (η_c = 1.015), equal to the sonic radius.

where m_i is A_i times the proton mass m_p; taking $Z_i/A_i \simeq 1/2$, $Z_i \simeq 10$, $r_c \simeq 500\ \mu$, and $n_c \simeq 10^{21}$ cm^{-3} (1.06 μ light) we obtain

$$\phi^* \simeq 2.3 \times 10^{14}\ \mathrm{W/cm^2}.$$

It appears that when a flux limiter[3] is needed and used the solution is modified, although quantitative corrections to the present results are small if the flux limiter lies within the range usually considered.

If Z_i is not large, the ion energy is not decoupled from the other equations, making the analysis more in*volved, although no large correction seems to follow.*

Figure 6 shows P_a normalized with the ablation pressure of steady, planar flow, using $W/4\pi r_a^2$, instead of $W/4\pi r_c^2$, in the expression for the laser intensity. As the power increases, the normalized pressure first drops slightly and then goes up very slowly, but always stays close to unity. Note that $P_a \sim n_c^{1/3}$ always. This result invalidates the hypothesis of Caruso[11] that, for $r_c/r_a \gg 1$, n_c would be an ignorable parameter. We find that for $r_c/r_a \gg 1$ *most* of the energy flows outward; this unexpected result explains the failure of Caruos's hypothe*sis, that leads to pressures much larger than those* found here. Estimates of ablation pressure,[12] assuming flux limited conduction, give values of P_a, $(r_c/r_a)^{2/3}$ times larger than found here.

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APPENDIX

The heat to internal-energy-convection flow ratio

$$\phi \equiv \frac{|\bar{K} T_e^{5/2} dT_e/dr|}{\frac{3}{2} n k T_e v}$$

is a measure of the nonisentropic character of the flow. It is easily verified, using Eqs. (27)–(29), that as $\eta \to \infty$,

$$\phi \sim \eta^{-7/3} \to 0, \quad \bar{W} < \bar{W}^*,$$

$$\phi \to 7/3, \quad \bar{W} = \bar{W}^*,$$

$$\phi \sim \eta^{2/7} \to \infty, \quad \bar{W} > \bar{W}^*.$$

Thus at $\bar{W} = W^*$, heat conduction goes discontinuously from negligible to dominant at vanishing density.

For $\eta \gg \eta_c > 1$, the Mach number u^2/θ is negligible and Eq. (12) becomes

$$\frac{du^2}{d\eta} = \frac{4\theta}{\eta} - \frac{2d\theta}{d\eta}. \qquad \text{(A1)}$$

Let us define phase-plane variables

$$\hat{M}^2 = (2\bar{W} - u^2)/\theta, \quad Y = \beta\eta\theta^{5/2};$$

then, from Eqs. (A1) and (14), we obtain

$$\frac{d\hat{M}^2}{dY} = \frac{(5 - \hat{M}^2)(1 - \hat{M}^2/2) - 4Y}{Y^2 + 5Y(5 - \hat{M}^2)/4}. \qquad \text{(A2)}$$

The asymptotic solutions given in Eqs. (27)–(29) are represented by the points $(Y=0, \hat{M}^2=5), (Y\to\infty, \hat{M}^2/Y=4/7), (Y=35/4, \hat{M}^2=12)$; the first two are saddle points of (A2) and the third is a node. For $\overline{W}<\overline{W}^*$, the solution, in the $(Y, \hat{M}^2)$ plane starts at $Y=0, \hat{M}^2=5$ (large η) and goes over to the node $Y=35/4$, $\hat{M}^2 \doteq 12$ (as η decreases). Similarly, for $\overline{W}>\overline{W}^*$, the solution comes from infinity $(\hat{M}^2/Y=4/7)$ toward the node as η decreases.

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